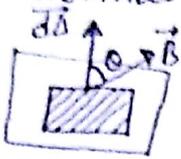


# ELECTROMAGNETIC INDUCTION [E.M.I.]

To produce induced current in the loop or induced emf, there should be relative motion between coil & the magnets, so that the magnetic flux change w.r.t. time.

## Magnetic flux ( $\phi$ )

It is no. of lines of force passing through a surface placed normal to magnetic field.



flux with small area element.

$$d\phi = B(dA \cos \theta)$$

$$d\phi = \vec{B} \cdot d\vec{A} \quad (\text{scalar quantity})$$

unit  $\rightarrow$  wb, Tm<sup>2</sup> = MKS

Maxwell, (10<sup>8</sup> cm<sup>2</sup>) = CGS

$$1 \text{ wb} = 10^8 \text{ Maxwell}$$

Case-I  $\rightarrow$  If field is uniform & surface is plane.

$$\phi_{\text{net}} = \int B dA \cos \theta$$

$$= B \cos \theta \int dA$$

$$* \phi_{\text{net}} = B A \cos \theta$$

$$\phi_{\text{net}} = \vec{B} \cdot \vec{A}$$

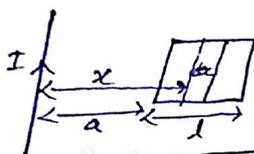
If no. of turn in the coil are 'n'

$$\phi = N B A \cos \theta$$

Case-II  $\rightarrow$  In case of non-uniform field.

$$* \phi_{\text{net}} = \int \vec{B} \cdot d\vec{A}$$

\*\* # A square of side 'l' is placed in same plane with a long wire as shown fig. then flux with loop.



$$\phi_{\text{net}} = \frac{\mu_0 I l}{2\pi} \log_e \left[ \frac{a+l}{a} \right]$$

\*\*\* #

$$\phi = N B A \cos \theta$$

$$\phi = f(B, A, \theta)$$

$\rightarrow$  If  $\phi \Rightarrow \text{const} \Rightarrow \text{NO EMI.}$   
 $\rightarrow$  If  $\phi \Rightarrow \text{non-const} \Rightarrow \text{EMI.}$

||  $\rightarrow$   $A \perp B \Rightarrow \phi = 0$

|||  $\rightarrow$   $\phi = 0$

iii) →



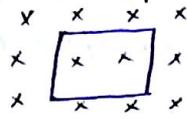
B wire  $\perp$  A coil  
 $\Rightarrow \phi = 0$

iv) →



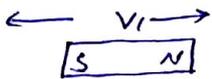
$\phi \neq 0$  but const  
 $\Rightarrow \frac{d\phi}{dt} = 0 \Rightarrow \text{NO EMI}$

v) → If a coil placed in uniform transverse magnetic field.



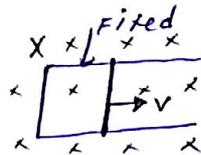
$\phi \neq 0$   
 but const.  
 NO EMI

vi) →



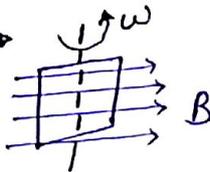
$\Rightarrow B \neq \text{const}$   
 $\phi \neq \text{const}, \text{EMI}(v)$

vii) →



$A \neq \text{const}$  EMI(v)  
 $\phi = \text{const}$

viii) →



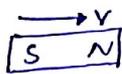
$\phi \neq \text{const}$  EMI(v)  
 $\theta \neq \text{const}$

ix) →



$\phi \neq 0$  NO EMI  
 $\phi = \text{const}$

x) →



$\phi \neq 0$  NO EMI  
 $\phi = \text{const}$

### # Faraday's Law of EMI

on the basis of his observation, Faraday given, a mathematical form of b/w the changing flux & Induced emf.  
 "Induced emf in a closed loop is always equal to Rate of change of flux w.r. + time"

$$* e = - \frac{d\phi}{dt}$$

If loop contains 'N' turns.

$$* e = - N \frac{d\phi}{dt}$$

$\phi = \text{magnetic flux}$

$e = \text{Induced emf in closed loop}$

**NOTE** \*  $\ominus$ ve sign indicates that change in flux always opposes the induced emf generated in the loop.

\*  $\phi = BA \cos \theta$  It can be changed with the variation of B, A  $\neq 0$ .

\* Induction takes place till the flux keep on changing.

$$|e| \propto \frac{d\phi}{dt}$$

11) → Variation w.r.t Varying Magnetic field

$$\phi = BA \cos \theta$$

$$e = -\frac{d\phi}{dt} = -\left\{ \frac{dB}{dt} \right\} A \cos \theta$$

Rate of change of magnetic field.

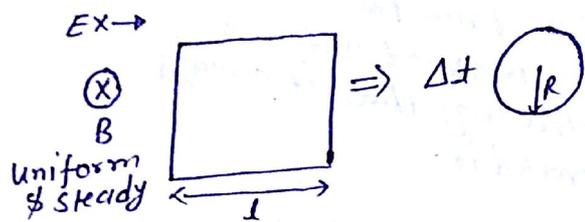
Uniform field

A uniform field is that which is same at all points in the space but its magnitude may change w.r.t time.

Steady field

Which is const. w.r.t time but it may be different at different points of the space.

12) → Variation w.r.t Varying Area of coil



It becomes circular in time 'Δt' then  
Average Induced EMF.

$$2\pi R = 4l$$

$$R = \frac{2l}{\pi}$$

$$\Delta e = B \left( \frac{\Delta A}{\Delta t} \right) \cos \theta$$

$$A_i = l^2$$

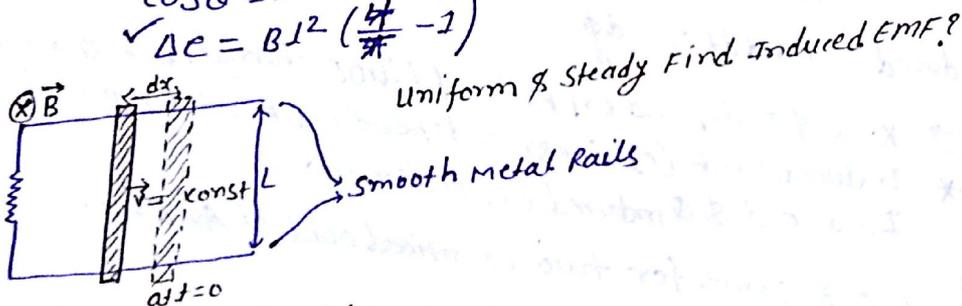
$$A_f = \pi R^2 = \pi \times \frac{4l^2}{\pi^2} = \frac{4l^2}{\pi}$$

$$\Delta A = l^2 \left( \frac{4}{\pi} - 1 \right)$$

$$\cos \theta = 1$$

$$\Delta e = B l^2 \left( \frac{4}{\pi} - 1 \right)$$

NCERT  
Ex →



$$v = dx/dt, dA = L dx$$

$$e = -B \frac{dA}{dt} \cos \theta = -B \times L \frac{dx}{dt} = -BLv$$

$$I_{\text{induced}} = \frac{e}{R} = \frac{BLv}{R}$$

Induced current & Total charge flow

$$e = -\frac{d\phi}{dt} = \left| \frac{d\phi}{dt} \right|$$

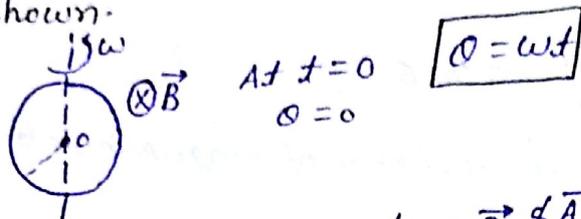
$$I_{\text{induced}} = \frac{e}{R} = \frac{1}{R} \left| \frac{d\phi}{dt} \right|$$

$$\text{Charge flow} = \Delta q = \frac{\Delta \phi}{R}$$

q → charge flow due to induced emf.

3] → Variation with varying angle b/w  $\vec{B}$  &  $\vec{A}$  i.e.  $\odot$   
 A coil is kept such that it is  $\perp$  to magnetic field

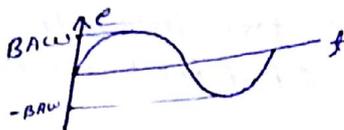
$\vec{B}$  as shown.



Let at  $t=t$ , the angle b/w  $\vec{B}$  &  $\vec{A}$  be  $\theta$

$$\phi = BA \cos \theta$$

$$\frac{d\phi}{dt} = BA(-\sin \theta) \frac{d\theta}{dt}$$



$$* e = -\frac{d\phi}{dt} = BA\omega \sin \omega t$$

→ This is the principle of AC generator.

### Lenz's Law

Lenz explain the  $\ominus$ ve sign of Faraday's law in accordance with energy conservation principle.

According to his law, induced emf in a close loop induced current in such a way that it always opposes the cause which has generated it.

### Formula for EMI

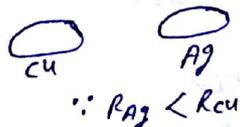
(i)  $\left\{ e = -\frac{d\phi}{dt} \right\}$

(ii)  $I_{\text{induced}} = \frac{e}{R} = -\frac{1}{R} \frac{d\phi}{dt}$

(iii)  $Q_{\text{induced}} = I_{\text{ind}} dt = -\frac{d\phi}{R}$

NOTE → \*  $e$  &  $I$  depend on Rate of flux change but  $Q$  doesn't depend.  
 \* Induced emf ( $e$ ) doesn't depend on resistant coil while  $I_{\text{induced}}$  &  $Q_{\text{induced}}$  on resistant.

EX →  $\frac{d\phi}{dt} \neq 0$  & same for two identical coil of Ag & Au.



ii) →  $\because e \propto R$   $\left[ e_{Cu} = e_{Ag} \right]$

iii) →  $I_{\text{induced}} \propto \frac{1}{R}$   $\left[ I_{Cu} < I_{Ag} \right]$

# In following coil  $B \neq \text{const}$

$$\Rightarrow \frac{d\phi}{dt} \neq 0$$

	Metal	plastic	Metal
EMI	✓	✓	✓
e	✓	✓	✓
I <sub>ind</sub>	✓	X	X

NOTE →

\* Induced emf is independent of Resistance & nature of coil (Metal, Plastic, Wood etc).

\* e & I depend on rate of flux change while Q<sub>induced</sub> depend flux change only.

Fast change of flux ⇒ High e & I  
 Slow change of flux ⇒ Low e & I  
 NO change of change ⇒ NO e & I

\* |e| ∝ V<sub>relative</sub>

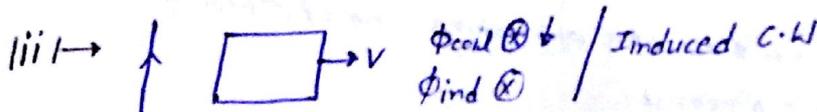
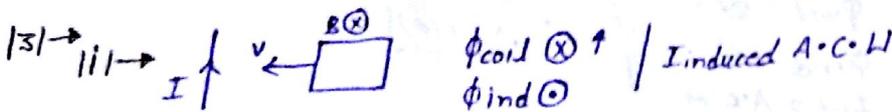
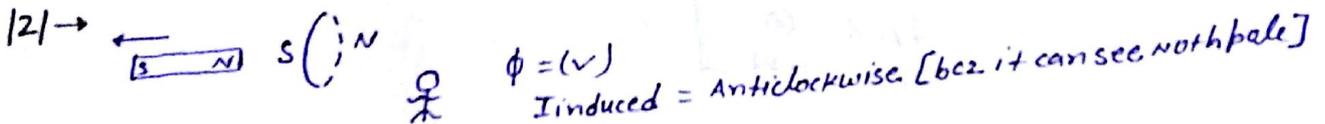
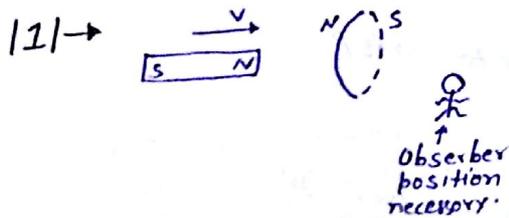
# Two circular coil are concentric & co-planar are  $N_1, r_1$  &  $N_2, r_2$  &  $r_1 \ll r_2$ . If current of coil two is changing at const rate  $\alpha$  amp/sec then calculate induced emf in coil one.



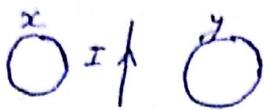
$$e = - \frac{\mu_0 N_1 N_2 \pi r_1^2 \alpha}{2 r_2}$$

\*\*\*

# Direction of Induced current using Lenz Law.



13/→



$\therefore \phi_x = \odot = \text{const}$   
 $\phi_y = \otimes = \text{const}$  } NO EMI

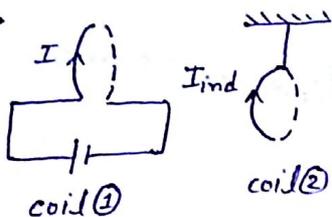
Case (i) → I wire. (+) ses.

	$\frac{x}{C}$	$\frac{y}{C}$
$\phi_{\text{coil}}$	$\odot \uparrow$	$\otimes \uparrow$
$\phi_{\text{ind}}$	$\otimes$	$\odot$
$I_{\text{induced}}$	C.W	A.C.W

Case (ii)  $I_{\text{ind}} (\downarrow)$  ses.

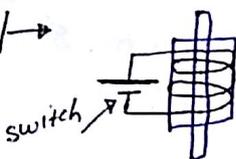
	$\frac{x}{C}$	$\frac{y}{C}$
$\phi_{\text{coil}}$	$\odot \downarrow$	$\otimes \downarrow$
$\phi_{\text{ind}}$	$\odot$	$\otimes$
$I_{\text{ind}}$	A.C.W	C.W

14/→



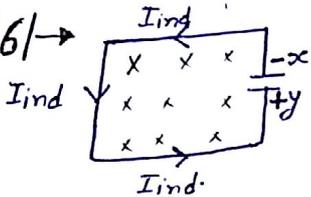
\* If coil 1 is heated  
 $R(\uparrow) \Rightarrow I(\downarrow) \Rightarrow B_1(\downarrow) \Rightarrow \phi_1(\downarrow)$  leftward.  
 So, Induced current in coil 2 is same direction coil 1  
 So magnetic attraction b/w the coil b/c current in same direction.

15/→



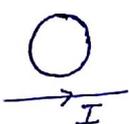
Flux with tiny loop is zero. If switch is made on suddenly some coil is associated with the loop & to oppose that opposite direction current is induced in tiny loop, so due to magnetic repulsion in more away (E24 & 25).

16/→



\* If field ( $\uparrow$ ),  $\phi_{\text{coil}} \uparrow$ ,  $\phi_{\text{ind}} \odot$ ,  $I_{\text{ind}} = \text{A.C.W}$   
 \* using this induced current capacitor get charged so plate x become  $\ominus$  & y  $\oplus$

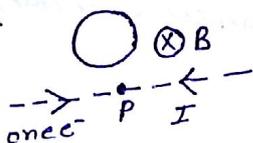
17/→



$I = \text{const.}$   
 $\phi_{\text{coil}} = 0 = \text{const.}$  } NO EMI

AIMS

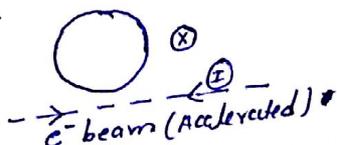
18/→



$\phi_{\text{coil}} = \uparrow \otimes$   
 $\phi_{\text{ind}} = \odot$   
 $I_{\text{ind}} = \text{A.C.W}$  } When  $e^-$  is towards P

$\phi_{\text{coil}} = \otimes \downarrow$   
 $\phi_{\text{ind}} = \otimes$   
 $I_{\text{ind}} = \text{C.W}$  } When  $e^-$  away from P

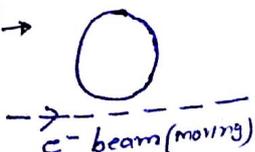
19/→



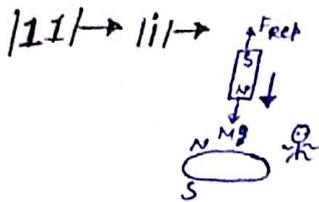
$\phi_{\text{coil}} = \otimes \uparrow$   
 $\phi_{\text{ind}} = \odot$   
 $I_{\text{ind}} = \text{A.C.W}$  }  $\text{Accel.} \neq 0$   
 $\boxed{V(\uparrow), I(\uparrow)}$

AIMS

10/→



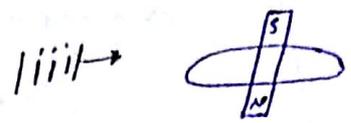
$I = \text{const.}$  & no. of  $e^-$  approaching = no. of  $e^-$  moving.  
 $\phi_{\text{coil}} = 0 = \text{const.}$   
 NO EMI



$I_{ind} = A \cdot c \cdot W = \nu =$  seen by observer

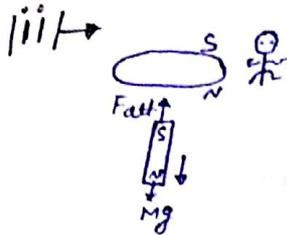
$F_{net} = mg - F_{rep}$

$F_{net} < Mg$   
 $a < g$



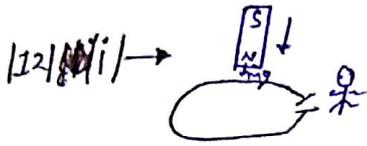
$I_{ind} = 0$   
At this instant Induced change the direction so

$I_{ind} = 0$   
 $a = g$



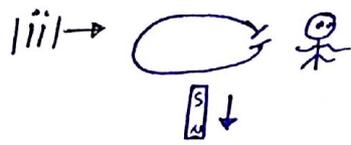
$c \cdot W = I_{ind}$   
 $F_{net} = mg - F_{att}$

$F_{net} < Mg$   
 $a < g$



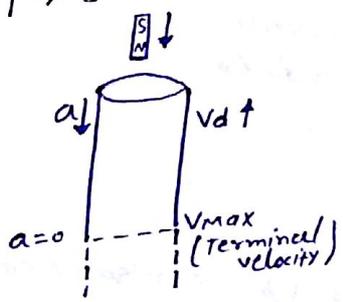
$\phi = \nu$   
 $I = x$  (no cross loop)  
 $EMF = \nu$

$I_{ind} = 0$   
 $a = g$



$e \neq 0$   
 $I_{ind} = 0$   
 $a = g$

\*\*\*  
|13| → Bar magnet falling in Long metal pipe.

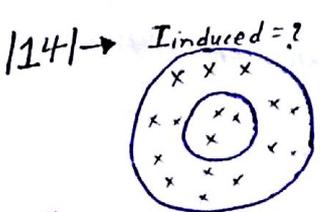


$a < g$   
 $v(t) = e(t) \Rightarrow I_{ind}(t)$   
opposition  $(t) = a(t)$



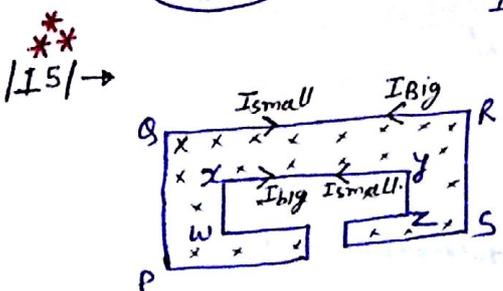
at last  $a = 0$  ( $v = v_{max}$ ) (Terminal velocity).

NOTE → \* If pipe is heated then its Resistance  $(\uparrow)$  so for balancing the Rate of  $e \propto v_{rel}$ , so final terminal velocity is more than previous case It is obtain at lower position in pipe than above case.



If  $B(t)$ , then for each coil

$\phi_{coil} \otimes \uparrow$   
 $\phi_{induced} \odot$   
 $I_{ind} = A \cdot c \cdot W$

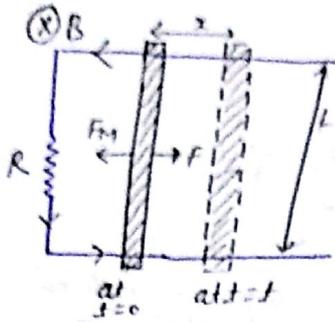


$(Area)_{big} > (Area)_{small}$   
 $(I_{ind})_{big} > (I_{ind})_{small}$

By superposition of current as shown in diagram.

$I_{QR} \neq 0$  (R to Q)  
 $I_{xy} \neq 0$  (x to y)

# Rod is moved with const. force ( $F$ ) as shown. Find its velocity as function of time.



$$F = ma = a = F/m$$

$$\text{Induced emf} \Rightarrow \boxed{e = BvL} \quad (\text{at } t=t)$$

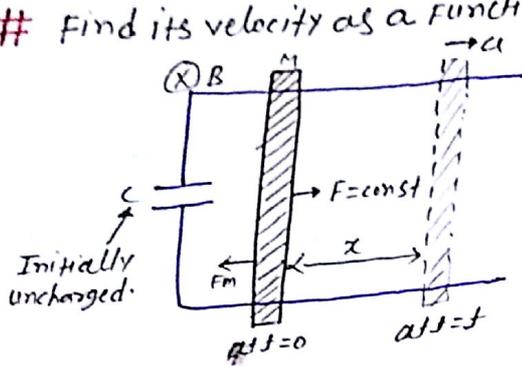
$$\text{Induced } = \frac{BvL}{R}$$

$$F_m = IBL = \frac{B^2 v L^2}{R}$$

At  $t \rightarrow \infty$

$$\boxed{V = \frac{FB}{B^2 L^2}} \rightarrow \text{Terminal velocity}$$

# Find its velocity as a function of time.



induced emf at  $t=t$

$$v = e = BvL$$

$$q = cv = cBvL$$

$$i = \frac{dq}{dt} = cBL \frac{dv}{dt}$$

$$\boxed{F_m = iBL = B^2 L^2 c \frac{dv}{dt}}$$

$$\boxed{a = \text{const.}}$$

$$\# \phi = f(B, A, \theta)$$

$$\text{If } \frac{d\phi}{dt} \neq 0 \Rightarrow \text{EMI}$$

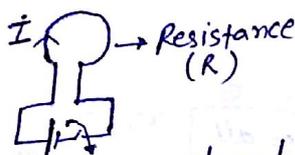
- Static EMI (If  $B \neq \text{const}$ )
  - MOTIONAL/DYNAMIC (If  $A \neq \text{const}$ )
  - Periodic EMI (If  $\theta \neq \text{const}$ )
- Self Induction  
 → Mutual Induction

### # Self Induction

- \* When current in a coil changes & Induction takes place in same coil.
- \* Due to self Induction coil opposes the change in current.

### Self Inductance (L)

It is a property due to which coil opposes the change of current. So, it is called "Inertia of Elasticity".



- (i) → Resistance of coil.
- (ii) → Induced emf in the coil.

Flux passing through the coil

$$(\phi_{\text{self}} \propto I)$$

$$\boxed{\phi_{\text{self}} = LI}$$

self induction of coil.

NOTE → Heber/Ampor Henry.

ii) → Induced emf in the coil  $(e) = -\frac{d\phi}{dt} = -L\left(\frac{dI}{dt}\right)$

\* During charging of current through a coil, there is an additional battery in the ckt whose emf can be written as-

emf of induced battery  $= L\left(\frac{dI}{dt}\right)$

\*  $R = \frac{V}{I}$

It is property which oppose the charge flow.

\*  $C = \frac{Q}{V}$

It is capacitance to hold the charge.

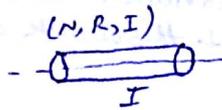
\*  $L = \frac{\phi}{I}$   
 self inductance of coil.  
 (oppose the change in current)

# 'L' circular coil →



$L_{\text{circular}} = \frac{\mu_0 \mu_r N^2 \pi R}{2}$

# 'L' solenoid



$L_{\text{solenoid}} = \mu_0 \mu_r n^2 \text{vol}$

# 'L' Toroid

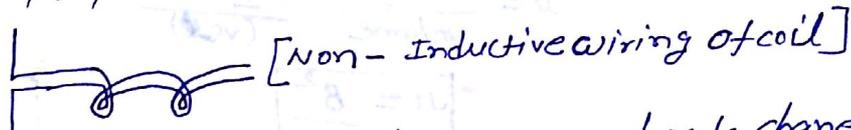


$L_{\text{toroid}} = \frac{\mu_0 \mu_r N^2 \delta_{\text{frame}}}{2 R_m}$

\* L → does not depend on  $\begin{cases} \phi \\ I \end{cases}$   
 \* L → depend on dimension parameter & number of turns & medium b/w the coil.

\* which of the following are possible.

- ii) →  $L=0$  &  $R=0$  (x)
- iii) →  $L \neq 0$  &  $R \neq 0$  (v) In practical choke coil.
- iiii) →  $L \neq 0$  &  $R=0$  (v) In ideal choke coil condn. only
- li) →  $L=0$  &  $R \neq 0$



→ In this arrangement is used in resistance box to cancel the 'L' of inside coil.

# Two circular coil  $L_1/L_2 = ?$  If →

ii) → same no. of turns radius  $R_1$  &  $R_2$  →  $\frac{L_1}{L_2} \propto \frac{R_1}{R_2}$

iii) → same radius & turn  $N_1$  &  $N_2$  →  $\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2$

iiii) → If circular coil of  $N_1$  turn converted into  $N_2$  turns.

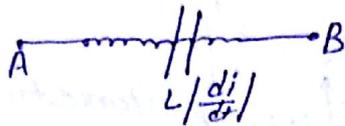
\*  $\frac{L_1}{L_2} = \frac{N_1}{N_2}$

li) → coil of radius  $R_1$  converted  $R_2$  radius

\*  $\frac{L_1}{L_2} = \frac{R_2}{R_1}$

## # Inductor

It is a long solenoid which behaves like source of emf when current through it changes.



\* If current is  $\uparrow$  through inductor

$$* V_A - V_B = L \left| \frac{di}{dt} \right|$$

\* If current is  $\downarrow$  through inductor

$$* V_A - V_B = -L \left| \frac{di}{dt} \right|$$

## Potential Energy (U)

For stabilising current in conductor external work is done b/c opposes change of current this work done is stored in the form of P.E

In Inductor \*

$$U = \frac{1}{2} LI^2$$

In capacitor \*

$$U = \frac{1}{2} CV^2$$

- \* This Energy is stored in the form of magnetic field.
- \* Energy spend by battery is  $LI^2$  so Remaining half part wasted in form of Heat.

## Energy density (u)

Energy stored per unit volume

$$u = \frac{\text{Energy}}{\text{volume}} = \frac{\frac{1}{2} LI^2}{(\text{vol})}$$

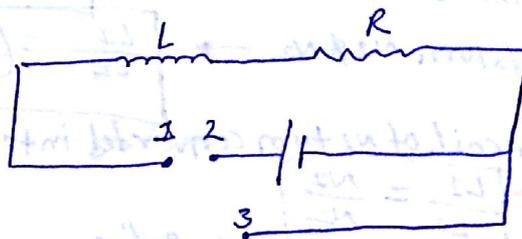
$$* u = \frac{B^2}{2\mu_0}$$

solenoid

---

$B = \mu_0 Ni$   
 $L = \mu_0 N^2 A$   
 'u' per unit  
 length =  $\frac{1}{2} (\mu_0 N^2 A) i^2$

## Charging & Discharging of Inductor



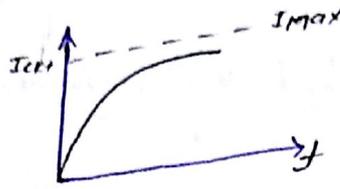
Case I → Charging or, Growth of current (1-2)

$t = 0$	$i(t)$	$t = \infty$ (full charge)
$e = \text{max}$	$e(t)$	$e = 0$
$I_{ckt} = 0$	$I_{ckt}(t)$	$I_{ckt} = \frac{V_0}{R} = \text{max}$
$U_L = 0$	$U_L(t)$	$U_L = \text{max}$



→ By KVL

$$\begin{cases} I_{ckt} = I_{\text{max}} (1 - e^{-\frac{R}{L}t}) \\ I_{ckt} = \frac{V_0}{R} (1 - e^{-\frac{R}{L}t}) \end{cases}$$



→ Time const  
Time in which current become approx 63% of  $I_{\text{max}}$ .

$$\tau = \frac{L}{R} \quad \begin{cases} t = \tau = 63\% \\ t = 5\tau = 99.3\% \end{cases}$$

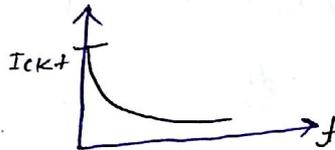
Case II → Discharging or, Decay of current (1-3)

$t = 0$	$i(t)$	$t = \infty$ (Full discharge)
$U_L = \text{max}$	$U_L(t)$	$U_L = 0$
$I_{ckt} = \text{max}$	$I_{ckt}(t)$	$I_{ckt} = 0$

\* During discharging induced current is in same direction as charging current

→ By KVL

$$I_{ckt} = I_{\text{max}} = e^{-\frac{R}{L}t}$$



→ Time const

$$\tau = \frac{L}{R} \quad \begin{cases} t = \tau = 63\% \text{ discharge} \\ t = 5\tau = 99.3\% \text{ discharge} \end{cases}$$

\* \*

In Inductor	In capacitor
* At $T = 0$ $L = \text{open ckt}$	* At $T = 0$ $C = \text{short ckt}$
* At $T = \infty$ $L = \text{short ckt}$	* At $T = \infty$ $C = \text{open ckt}$

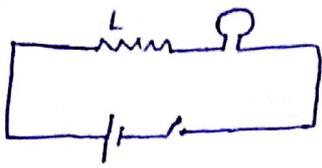
NOTE → \* If Inductor (L) are in series

$$L_{\text{eff}} = L_1 + L_2 + L_3 + L_4 + \dots + L_n$$

\* If Inductor (L) are in parallel.

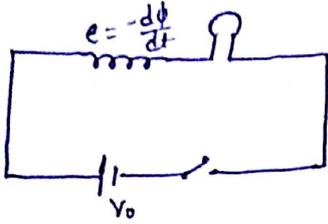
$$\frac{1}{L_{\text{eff}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \dots + \frac{1}{L_n}$$

NCERT  
#



When switch is made on current ( $\uparrow$ ) se exponentially  
So bulb brightness ( $\uparrow$ ) se gradually & acure max brightness  
After some time.

NCERT  
#



ii)  $\rightarrow$  When switch is ON.

$$\phi(\uparrow) \Rightarrow \frac{d\phi}{dt} = -ve \Rightarrow e = -ve$$

$$\Rightarrow V_{bulb} = V_0 - e$$

ii)  $\rightarrow$  After switch made of  
 $\phi(\downarrow) \Rightarrow \frac{d\phi}{dt} = +ve$

$$e = +ve$$

$$\Rightarrow V_{bulb} = V_0 + e$$

So bulb may not get fused or  
sparking takes place near  
the switch.

iii)  $\rightarrow$  After some time

$$I = \text{const}$$

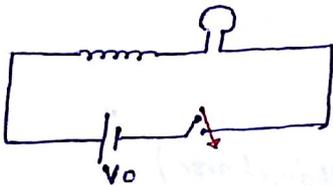
$$\phi \leftrightarrow$$

$$\frac{d\phi}{dt} = 0$$

$$e = 0$$

$$V_{bulb} = V_0 \text{ (proper bright)}$$

NCERT  
AIIMS  
#



\* During insertion of IRON Rod in coil

$$\phi(\uparrow) \Rightarrow \frac{d\phi}{dt} = +ve$$

$$e = -ve$$

$$V_{bulb} = V_0 - e \Rightarrow \text{brightness} \downarrow$$

\* After full insertion flux become MAX but const so no EMI.

$$V_{bulb} = V_0 \text{ (bulb Acuire its initial max brightness)}$$

NOTE  $\rightarrow$  Battery equivalent model

ii)  $\rightarrow$  If current is ( $\uparrow$ ) then

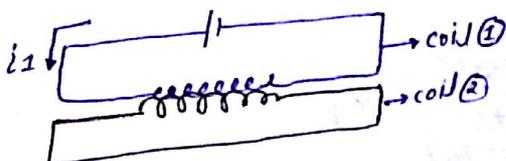
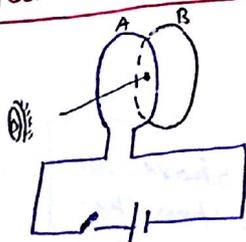
$$|e| = L \frac{dI}{dt}$$

iii)  $\rightarrow$  If current is ( $\downarrow$ ) then

$$|e| = L \frac{dI}{dt}$$

# Mutual Induction (M)

Ability to Resist change in flux due to change in  
current in the other coil is defined by mutual  
Inductance of the system.



Flux passing through coil 1 due to  
current  $i_1$  in it  
 $\phi_1 \propto i_1$

Some part of  $\phi_1$  will also linked  
with coil 2 say  $\phi_{21}$

$$\phi_{21} \propto i_1$$

$$* \phi_{21} = M_{21} i_1$$

M depend on  $\rightarrow$

- \* Shape & size of coil.
- \* Dist. b/w coils.
- \* Medium b/w coil.
- \* Orientation b/w coil.

$M_{21}$  = mutual inductance of coil (2) w.r.t coil (1)  
 Similarly, If coil (2) is connected with battery instead of coil (1)

$$\phi_2 \propto i_2$$

$$\phi_2 \propto i_2$$

$$\boxed{\phi_2 = M_{12} i_2}$$

$M_{12}$  = mutual inductance of coil (1) w.r.t coil (2)

\* For all type of geometry \*  $\boxed{M_{21} = M_{12}}$

\* scalar

\* unit  $\rightarrow \frac{Wb}{A}$ , Henry.

\*  $\rightarrow$  Induction emf due to mutual induction

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt} (M i_1)$$

$$\begin{aligned} * e_2 &= -M \frac{di_1}{dt} \\ e_1 &= -M \frac{di_2}{dt} \end{aligned}$$

\* M b/w two coil

$$\boxed{M = k \sqrt{L_1 L_2}}$$

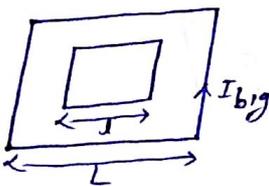
$L_1 \& L_2 \Rightarrow$  self induction of coil  
 $k \Rightarrow$  coupling coefficient.

$$\boxed{0 \leq k \leq 1}$$

\* Ideal or, perfect coupling ( $k=1$ ).

\* M is defined for a particular arrangement of two coil.

\*\*# Two square coil of side 'l' & 'l' are concentric & coplaner shown. If ( $L \gg l$ ). Then mutual induction?



$$* B_{big} = 2\sqrt{2} \frac{\mu_0 I_{big}}{\pi L}$$

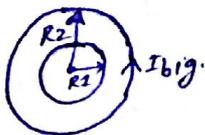
Flux with small coil

$$\phi_{small} = B_{big} A_{small} \cos 0^\circ$$

$$* \phi_{small} = \left( \frac{2\sqrt{2} \mu_0 I_{big}}{\pi L} \right) l^2$$

$$* M = \frac{\phi_{small}}{I_{big}} = \frac{2\sqrt{2} \mu_0 l^2}{\pi L}$$

\*\*# Two concentric coplaner circular loop as shown ( $R_1 \ll R_2$ ) Find M=?



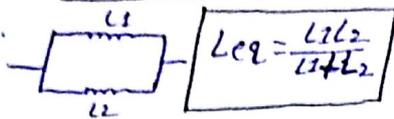
$$* I_{big} = \frac{\mu_0 I_{big}}{2R_2}$$

$$* \phi_{small} = \frac{\mu_0 I_{big}}{2R_2} (\pi R_1^2)$$

$$* M = \frac{\phi_{small}}{I_{big}} = \frac{\mu_0 \pi R_1^2}{2R_2}$$

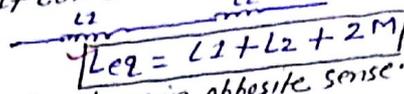
## Grouping of Inductor

ii) → Parallel

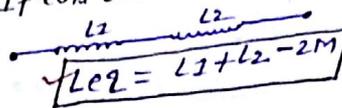


iii) → Series

a) → If coils are in same sense.

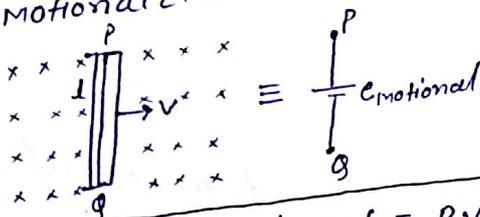


b) → If coils are in opposite sense.



## # Motional / Dynamic EMI

When a conducting rod moves in  $\perp$  magnetic field cutting the field line then due to magnetic force its free  $e^-$  are shifted at one end so potential difference is produced b/w the end of rod. This is called Motional EMI



∴ In steady state  $e\vec{E}_{induced} = e(\vec{v} \times \vec{B})$

$$E_{induced} = Bv$$

$$* V_p - V_q = BvL$$

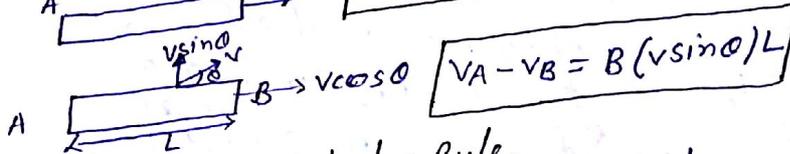
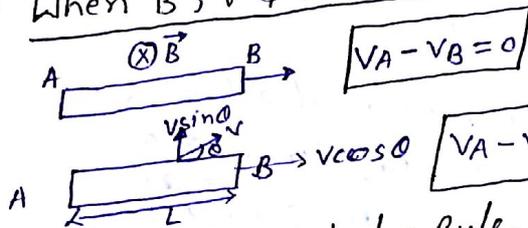
$$\text{Motional emf} = BvL$$

NOTE → \* Where, all the three vectors i.e.  $\vec{B}$ ,  $\vec{v}$  &  $\vec{L}$  are mutually  $\perp$ .

\* If all three vectors are coplaner or any two vector are parallel or anti-parallel then, no flux cutting. So,  $e_{motional} = 0$ .

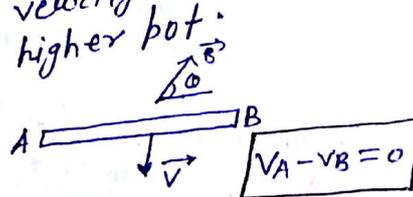
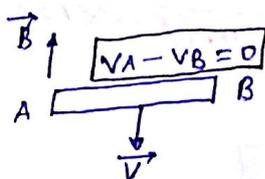
\* If  $\vec{v} \parallel \vec{B}$ ,  $\vec{B} \parallel \vec{L}$ ,  $\vec{L} \parallel \vec{v}$  or, all are coplaner then emf will be zero.

Case-I → When  $\vec{B}$ ,  $\vec{v}$  &  $\vec{L}$  are in different direction.

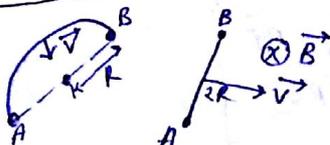


Right hand palm Rule

Finger → magnetic field.  
Thumb → velocity.  
palm → higher pot.



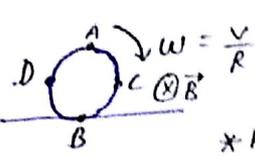
Case-II → Arbitrary shaped conductor



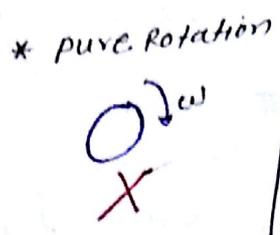
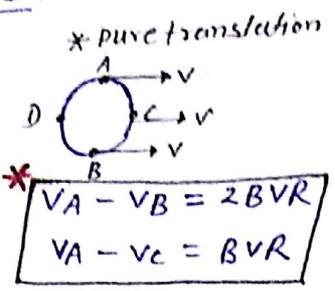
$$VA - VB = Bv(2R)$$

\*\*\*  
##

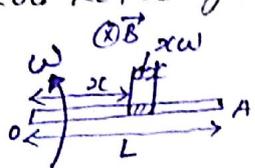
A Ring is pure rolling in magnetic field  $\vec{B}$ .



$v_A - v_B = ?$   
 $v_A - v_C = ?$



case III → Rod rotating about an end in  $\vec{B}$



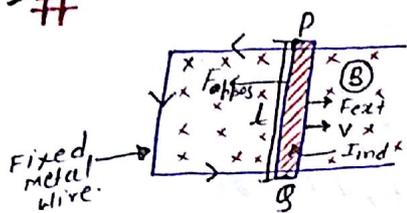
$$v_0 - v_A = \frac{1}{2} B \omega L^2$$

P.D b/w two points:

$$e = \int_{x_i}^{x_f} B \omega \pi dx$$

$$e = \frac{1}{2} B \omega (x_f^2 - x_i^2)$$

\*\*\*  
##



iii → If resistance of loop is 'R'

$$I_{ind} = \frac{e}{R} = \frac{Bvl}{R}$$

ii → opposing force on rod

$$F_{opps} = \left(\frac{Bvl}{R}\right) l B \sin 90^\circ$$

$$F_{opps} = \frac{B^2 l^2 v}{R}$$

iii → Required external force to maintain const velocity on rod

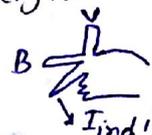
$$F_{ext} = F_{opps} = \frac{B^2 l^2 v}{R}$$

iv → Required ext. power to maintain const. velocity on rod.

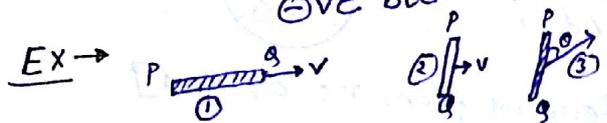
$$P_{ext} = \vec{F}_{ext} \cdot \vec{v}$$

$$P_{ext} = \frac{B^2 l^2 v^2}{R}$$

# Fleming Right hand Rule for direction of  $I_{induced}$  in moving Rod



\* This Rod is equivalent to cell so, current is not from ⊕ve to ⊖ve but is from ⊖ve to ⊕ve.

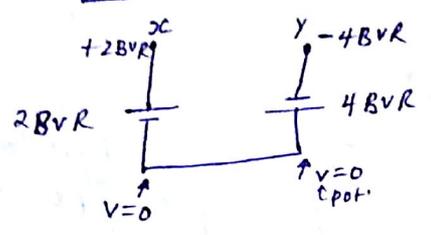
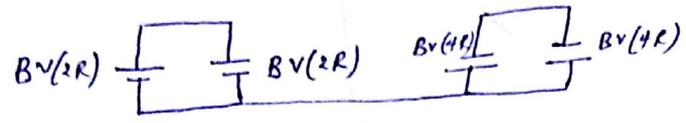
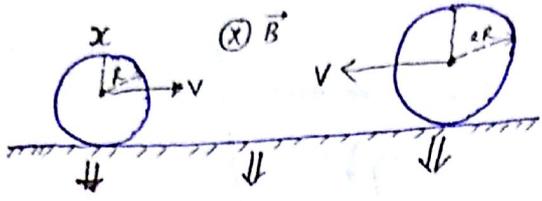


ii →  $\vec{J} \parallel \vec{v} \Rightarrow e = 0$  ( $\because \vec{J} \parallel \vec{v}$ , so no flux cutting)

iii → Flux cutting is so  $e_{motional} = Bvl$   
Induced = P to Q [Q = ⊖ve, P = ⊕ve]

iii → Flux cutting is there so  $e_{motional} = B(v \sin \theta)l$   
 $e = Bvl \sin \theta$   
Q = ⊖ve / P = ⊕ve.

# Two conducting Ring are moving linearly on a conducting surface as shown. Find potential difference b/w their Highest points?



$$V_x - V_y = 2BVr - (-4BVr)$$

$$V_x - V_y = 6BVr$$

# Free Falling metal Rod (North-south).  
\* Doesn't cut BH & Bv line so (emotional = 0)

# Free Falling metal Rod (East-West)  
\* Doesn't cut Bv line so (e = 0)

\* It cut BH line so (emotional = BHv)

# An Aeroplane moving in Horizontal plane at a fix height from the surface.  
\* V || l of aeroplane, so across the length of Aeroplane (e = 0)

\* Wings cut the Bv lines, so P.D across the wing e = Bv l wings.

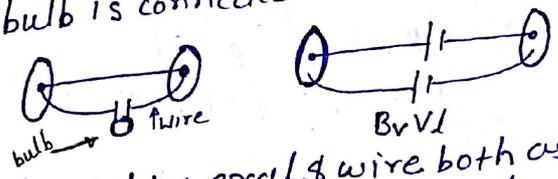
\* If ~~plane~~ plane is on equator Bv = 0, so PD across wings (e = 0)

# For moving train.

ii) → velocity of train || length of train, so PD across initial & Final End of train = 0.

iii) → Axial of train cuts Bv lines so, PD across axial e = Bv l axial.

\* iii) → If a bulb is connected b/w Ends of axial then,



There is P.D b/w axial & wire both as shown but as a loop net emf = 0 so, bulb does not glow.

# IF 'n' identical Rod is Rotates as shown.

\* Each Rod is equivalent to shell of  $\frac{1}{2} B\omega L^2$   
\* If there free end are connected by conducting wire as shown it is || grouping of 'n' identical cell.



ii) → P.D b/w Periferi & centre [Independent from no. of Rod]

$$= \frac{1}{2} B\omega L^2$$

iii) → P.D b/w two points of periferi = 0  
EX → Bicycle wheel.

# If no. of rods become infinite then it is equivalent to rotating disc  $\perp$  magnetic field.

So, PD b/w periferi & centre.

$$* PD = \frac{1}{2} B \omega R^2 \quad (R = \text{radius of disk})$$

NOTE  $\rightarrow$  Faraday copper disc <sup>generation</sup> based on this concept.  
Angular disc

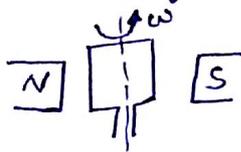
OF Rotating is Rotating is far field.

$$* P.D = \frac{1}{2} B \omega (R^2 - r^2)$$

\* Rotating disc, If metallic rod rotating  $\perp$  as disc as shown then non induction takes place. so  $PD = 0$

## # Periodic EMI

When a coil placed in magnetic field rotate an about axis shown. then its orientation w.r.t field changing continuously. so flux with coil changes continuously & induction takes place this is called periodic EMI.



Flux with coil ( $\phi$ ) =  $NBA \cos \omega t$

$\therefore$  Induced emf  $e = -\frac{d\phi}{dt} = -\frac{d}{dt} (NBA \cos \omega t)$

$$e = -NBA \frac{d}{dt} (\cos \omega t)$$

$$* e = NBA \omega \sin \omega t$$

## # Induced Electric Field

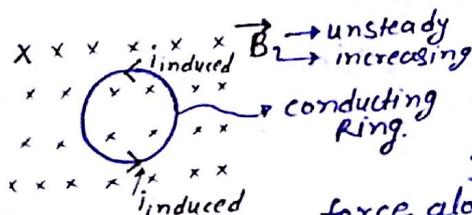
A time varying magnetic field produces an induced electric field which have following properties.

|a|  $\rightarrow$  They always exist in the form of closed loop may be circular or non-circular. (Work done in close loop is zero)

|b|  $\rightarrow$  They are non-conservative in nature.

$$i.e. \oint \vec{E} \cdot d\vec{l} \neq 0$$

|c|  $\rightarrow$  Their approximate direction can be found a/c to Lenz's Law.  
consider a time varying magnetic field & a circular Ring is kept inside the field.



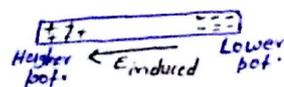
$$\oint \vec{E} \cdot d\vec{l} \neq 0$$

Initially all the  $e$ 's were at rest & magnetic force alone can't circulate them in the loop. so an induced electric field which is produced as a result of in steady magnetic field develops the induced current in the loop.

$\rightarrow$  In Electrostatics,

$$q \oplus \xrightarrow{E} \ominus q$$

But, Induced Electric field.

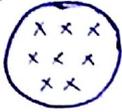


Faraday has measured the same  $\oint \vec{E} \cdot d\vec{l}$  which comes equal to  $-\frac{d\phi}{dt}$

$$\mathcal{E}_{\text{induced close loop}} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$$

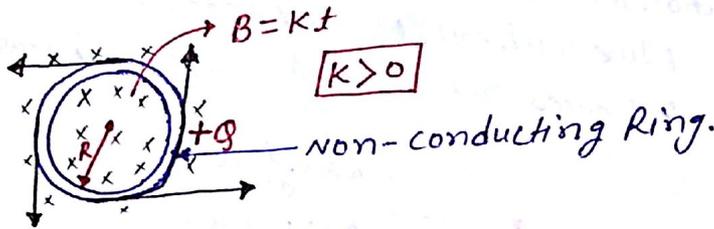
### Special case

$$B = f(t)$$



consider a case when magnetic field is confined in a cylindrical region as shown, if magnetic field is varying with time, induced electric field in the form of close circular loop will be developed around the region.

- # Find  
 |a| → Torque Acting on the Ring.  
 |b| → Initial Angular Acc<sup>n</sup>.



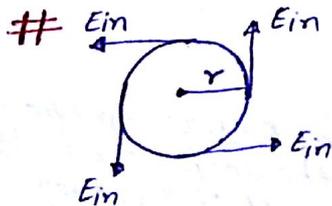
$$\int d\tau = \int \{(dq) E\} R$$

$$\tau = QER = Q \left[ \frac{R}{2} k \right] R$$

$$\tau = \frac{QkR^2}{2}$$

$$I\alpha = \frac{kQR^2}{2}$$

$$\alpha = \frac{kQ^2}{2M}$$



$$E_{in} = \begin{cases} -\frac{r}{2} \left( \frac{dB}{dt} \right), & r < R \\ -\frac{R^2}{2r} \left( \frac{dB}{dt} \right), & r \geq R \end{cases}$$

# A uniform  $\vec{B}$  is in cylindrical region of radius 'r' whose cross section is as shown. If field is changing at const. rate  $d\phi/dt$ . Find induced EMF field at position 'r' where.

ii)  $r > R$     iii)  $r < R$ .

ii)  Induction  $\oint \vec{B}$  in same coil ( $A = \pi R^2$ )

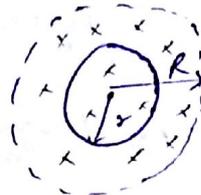
$$E \int dl = A \frac{dB}{dt}$$

$$E(2\pi r) = (\pi R^2) \left( \frac{dB}{dt} \right)$$

$$E \propto \frac{1}{r}$$

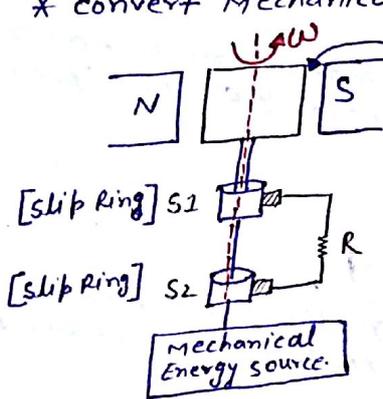
iii)  $\oint \vec{E} \cdot d\vec{l} = A \frac{dB}{dt}$

$$E(2\pi r) = (\pi r^2) \frac{dB}{dt}$$

$$E \propto r$$


### \*\* AC Generator

- \* Based on EMI (Periodic Induction)
- \* Convert mechanical energy into electric energy.



\* Flux at any instant 't'

$$\phi = NBA \cos \theta$$

\* Induced emf

$$e = - \frac{d\phi}{dt}$$

$$e = NAB\omega \sin \omega t$$

\* If resistance of loop 'R'

$$I_{ind} = \frac{e}{R}$$

$$I_{ind} = \frac{NAB\omega \sin \omega t}{R}$$

#  $\frac{e_{max}}{\phi_{max}} = ?$      $\frac{NAB\omega}{NAB} = \omega$

[cos leading 90° than sin]

\* Flux is leading emf by 90°, so when flux is max. then emf is zero.

\* Frequency of produced AC

$$F = \frac{N \times P}{2}$$

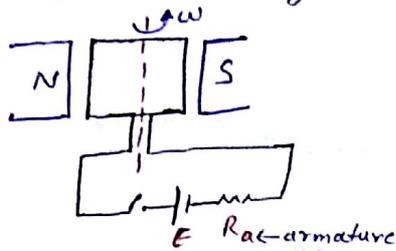
P = pole

N = No. of revolution per sec.

NOTE → In DC generator commutator is used in place of slip ring which makes the load current unidirectional.

# DC MOTER

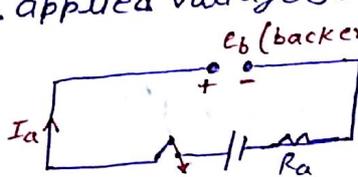
\* It convert Electric Energy into Heat energy. so, Its just opposite to generator.



- \* Due to current in coil torque is exerted on coil by external magnetic field so, coil rotate hence mechanical energy is produced.
- \* Due to flux change, back emf is produced in coil which oppose the applied voltage so equivalent ckt. of moter.

$$e = NAB\omega \sin \omega t$$

$$e \propto \omega$$



$$I = \frac{E - e_b}{R_a}$$

$$E = e_b + I_a R_a$$

Motor equation.

When moter is started?

- ii)  $t = 0 \Rightarrow \omega = 0 \Rightarrow e_b = 0 \Rightarrow I_a = \frac{E}{R_a}$ ,  $I_a = \text{max}$  & unsafe.
- iii)  $t \Rightarrow \omega \Rightarrow e_b \Rightarrow I_a \downarrow \Rightarrow$  moving towards safe current.
- iiii) After some moment,  $\omega = \text{const} \Rightarrow e_b \text{ const} \Rightarrow I_a \Rightarrow \text{const. (min \& safe)}$

Starter

It is variable resistance used to protect the moter from initial excess current as moter get speed its resistance get  $\downarrow$  & dt net speed it resist a zero resistance in moter ckt.

Power

$$* P_{\text{input}} = E I_a$$

$$* P_{\text{loss}} = I_a^2 R_a$$

$$* P_{\text{output}} = P_{\text{input}} - P_{\text{loss}}$$

$$= E I_a - I_a^2 R_a$$

$$= I_a [E - I_a R_a]$$

$$* P_{\text{output}} = I_a E_b$$

100% Efficiency not possible bcoz  $e_b = E$  not possible in this case current will be zero.

\* For max output power.

$$\frac{E e_b - e_b^2}{R_a}$$

$$\frac{dP_{\text{out}}}{de_b} = \frac{E - 2e_b}{R_a} = 0$$

$$\text{ii) } e_b = \frac{E}{2}$$

$$\text{iii) } \% \eta = \frac{e_b}{E} \times 100 = 50\%$$

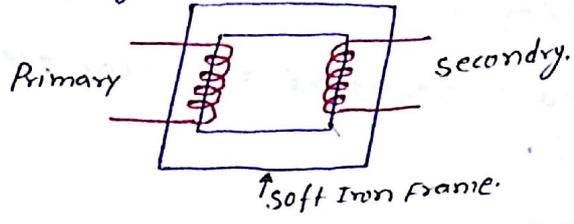
$$\text{iiii) } I_a = \frac{E - e_b}{R_a} = \frac{E}{2R_a}$$

$$\text{v) } (P_{\text{out}})_{\text{max}} = I_a e_b$$

$$= \left(\frac{E}{2R_a}\right) \left(\frac{E}{2}\right) = \frac{E^2}{4R_a}$$

# Transformer

- \* Based on mutual induction.
- \* Works for AC only not for DC.
- \* If (t) or (b) AC voltage & makes opposite change in current.
- \* Input frequency = output frequency.
- \* Phase difference of  $(\pi)$  b/w input & output voltage.
- \* convert high Alternating voltage to Low Alternating voltage & vice-versa.



\* For Ideal transformer power = const = VI = const.  $\therefore V \propto \frac{1}{I}$

$$K = \frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{\sqrt{Z_s}}{\sqrt{Z_p}}$$

secondary.                      primary.

Step up	Step down
$V_s > V_p$	$V_s < V_p$
$N_s > N_p$	$N_s < N_p$
$I_s < I_p$	$I_s > I_p$
$Z_s > Z_p$	$Z_s < Z_p$
$K > 1$	$K < 1$
↑ Turn Ratio more than one so, step up.	↑ Turn Ratio less than one so step down.

$\therefore R = \frac{l}{A} \propto \frac{1}{A}$

\* secondary wire will be thicker than primary wire.

\* Turn Ratio equal to 100. If 5v battery or cell connected at primary then secondary battery is source of DC so output = 0.

$$\% \eta = \frac{P_{out}}{P_{input}} \times 100$$

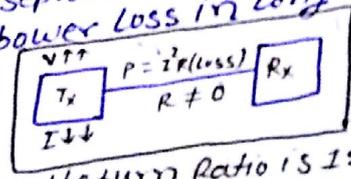
There is no moving part in this machine so, its efficiency more than generator.

## LOSS in transformer

- i) Flux loss → To minimise this soft iron frame is used.
- ii) Hysteresis loss → To minimise this Area of Hysteresis loop should be induce so soft iron is used.
- iii) Eddy current loss (Iron loss) → To minimise this laminated core of soft iron is used.
- iv) copper loss ( $I^2R$  loss) → Heat loss in copper binding.
- v) Humming loss → It is noise produced due to 'Magnetostriction effect'.

## Application of Transformer

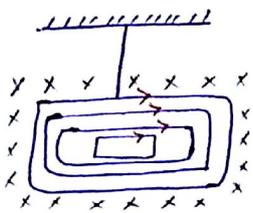
- \* For electrical separation b/w two ckt.
- \* use to reduce power loss in long distance power transmission.



**T/c #** In a setup transformer, the turn ratio is 1:2. A Leclanche cell (emf = 1.5V) is connected across the primary. Then voltage develop in the secondary coil will be → zero

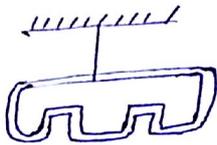
\* [Transformer doesn't work on dc]

# Eddy current



When flux with a metallic str. (plate, cylinder, sphere) changes with time, then induced currents are in the form of large no. of concentric loop. Which is similar to eddies in water so called Eddy current / Foucault current.

- ii) → Eddy current produce large amount of Heat.
- iii) → To reduce eddies current Resistance of metallic str. is (↑) using laminated or, slooted str.



By making slooted  $e^-$  is elongated. so Resistance (↑) so eddy current (↓).

## Application of Eddy current

### Dead beat (⊙)

In moving coil (⊙) vibrate for sometime at deflected position & dumping takes some time. In dead beat (⊙) inside a coil a metallic plate is inserted in which eddy current are produced due to vibration so it is produce fast damping.

- EX →
- \* Speed meter in vechile
  - \* Electric break in train
  - \* Induction cooker (Furness)
  - \* Induction coil in T.V
- learn in NCERT